Suppose we have two coordinate systems, x^μ and $x^{\mu'}$. We can change the basis in which the derivative is expressed as follows:

$$\frac{\partial f}{\partial x^\mu} = \frac{\partial f}{\partial x^{\mu'}} \frac{\partial x^{\mu'}}{\partial x^\mu} \Longleftrightarrow \partial_\mu = \frac{\partial x^{\mu'}}{\partial x^\mu} \partial_{\mu'} .$$

We recall that the coordinate basis for the tangent space (the set of all directional derivatives at a point $p \in M$) is given by $\{\partial_\mu\}_\mu$, such that any vector $V \in T_p$ can be written as the <u>operator</u>

$$V = V^\mu \partial_\mu$$

(recall that directional derivatives map functions on the manifold to the reals.)

If we change to the $x^{\mu'}$ coordinates, we have

$$V = V^\mu \partial_\mu = V^\mu \frac{\partial x^{\mu'}}{\partial x^\mu} \partial_{\mu'} \overset{!}{=} V^{\mu'} \partial_{\mu'} \Longleftrightarrow$$

$$\boxed{V^{\mu'} = \frac{\partial x^{\mu'}}{\partial x^\mu} V^\mu}$$