Escape Velocity

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Exercise:

We compute the escape velocity from the surface of a planet with mass M and radius R.

Solution:

We wish to find the initial radial velocity of a test mass such that it "just" escapes to infinity (i.e., $v(\infty) = 0$). We will suppose that energy is conserved once the test particle begins moving, and that the particle has mass m.

Energy conservation gives us

$$T_0 + V_0 = T_f + V_f = V_f.$$

By convention, the gravitational potential energy of two masses is defined to be 0 when they are infinitely separated, so we then have $T_0 + V_0 = 0$.

The gravitational potential energy is defined as the energy required to prepare the initial configuration by bringing in the masses from infinity. To bring the planet in the absence of another mass requires no work. Once the planet is in place, the work required to bring in the test mass is given by

$$W = -\int_{\text{path}} \vec{F} \cdot d\vec{r} = \int_{\infty}^{R} \frac{GMm}{r^2} dr = -\frac{GMm}{R}.$$

It follows that

$$\frac{1}{2} m v_{\text{esc}}^2 = \frac{GMm}{R} \implies \boxed{v_{\text{esc}} = \sqrt{\frac{2GM}{R}}.}$$