

# Fourier's Trick

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**Exercise:**

We derive Fourier's trick.

**Solution:**

We suppose we have a complete, orthonormal set of functions,  $\{\psi_n(x)\}_n$ . As the functions are complete, we can expand an arbitrary function  $f(x)$  in this basis:

$$f(x) = \sum_n c_n \psi_n(x).$$

We now solve for the coefficients  $c_n$ .

We take the inner product with a particular state  $\psi_m(x)$ :

$$\begin{aligned} \langle \psi_m | f \rangle &= \int \psi_m^*(x) f(x) dx = \int \psi_m^*(x) \sum_n c_n \psi_n(x) dx = \sum_n c_n \int \psi_m^*(x) \psi_n(x) dx = \\ & \sum_n c_n \delta_{nm} = c_m \implies \boxed{c_n = \langle \psi_n | f \rangle = \int \psi_n^*(x) f(x) dx.} \end{aligned}$$