Exercise: We show that gaps between successive primes can be arbitrarily large.

(Source: Terry Tao — Small and Large Gaps Between the Primes)

Proof

Let $n \in \mathbb{Z}_{>1}$. Let $z_i = n! + i$, for $i \in \{2, \dots, n\}$.

For all $i \in \{2, \dots, n\}$, $n! = 2 \cdot 3 \cdot \dots \cdot (i-1) \cdot i \cdot (i+1) \cdot \dots \cdot n$.

Thus, each z_i is composite, as we can write it

$$z_i = i\left(\frac{n!}{i} + 1\right) \in \mathbb{Z}.$$

We have therefore shown that, for arbitrary n, we can construct a sequence of consecutive composite numbers of length $n-1$, $\{z_i\}_{i=2}^{n}$.

By considering arbitrarily large n, we can therefore construct arbitrarily large gaps between primes. $\blacksquare$