First, we derive the formula for the partial sum

$$S_n = \sum_{j=0}^{n-1} x^j$$

We consider $S_n - xS_n = S_n(1-x) = 1 - x^n$,
where the final equality can be seen from using the definition of x_n.

It follows that

$$S_n = \frac{1-x^n}{1-x}$$

Then, if $|x|<1$, $\displaystyle\sum_{j=0}^{\infty} x^j = \lim_{n\to\infty} \frac{1-x^n}{1-x} = \boxed{\frac{1}{1-x}}$