We derive the maximum likelihood estimator for the temperature for the Maxwell-Boltzmann speed distribution in d-dimensions.

The Maxwell-Boltzmann speed distribution in d dimensions is given by

$$p_d(v) = \frac{1}{2^{-1+d/2}\Gamma(d/2)\gamma^{d/2}} v^{d-1} e^{-v^2/2\gamma},$$

where $\gamma = k_B T/m$. We compute the maximum likelihood estimator for γ.

Suppose we have n data points $\{v_i\}_{i=1}^n$ sampled $i.i.d.$ from $p_d(v)$. The likelihood of the dataset is given by

$$p_d(v_1, ..., v_n|\gamma) = \prod_{i=1}^{n} p_d(v_i|\gamma),$$

and therefore the log-likelihood is given by

$$\sum_{i=1}^{n} \log p_d(v_i|\gamma) = \sum_{i=1}^{n} \log \left[\frac{1}{2^{-1+d/2}\Gamma(d/2)\gamma^{d/2}} v_i^{d-1} e^{-v_i^2/2\gamma} \right]$$
$$= \sum_{i=1}^{n} \left[-\log \left(2^{-1+d/2}\Gamma(d/2) \right) - \frac{d}{2}\log(\gamma) - (d-1)\log(v_i) - \frac{v_i^2}{2\gamma} \right].$$

We now differentiate this expression with respect to γ to find the maximum likelihood estimate:

$$0 = \sum_{i=1}^{n} \left(-\frac{d}{2\gamma} + \frac{v_i^2}{2\gamma^2} \right) \iff \boxed{\hat{\gamma} = \frac{1}{d}\left(\frac{1}{n}\sum_{i=1}^{n} v_i^2 \right) = \frac{1}{d}\overline{v^2}}.$$

Recalling the definition γ, we conclude that the MLE estimate for the temperature according to the Maxwell-Boltzmann speed distribution is simply the empirical approximation of the equipartition theorem for a free particle in d-dimensions, $d \cdot \frac{k_B T}{2} = \frac{1}{2} m \langle v^2 \rangle$.