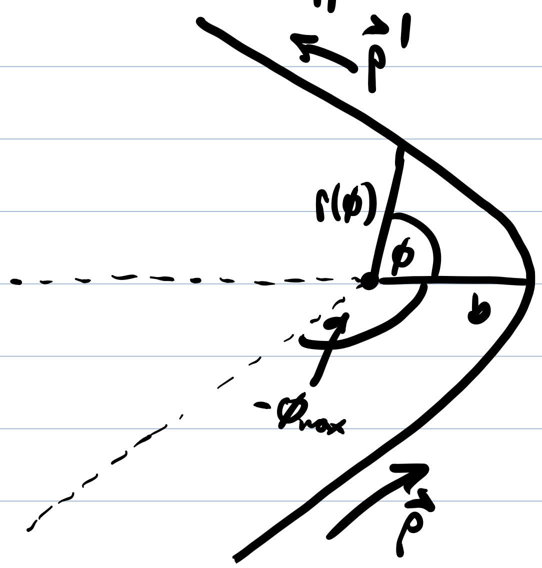
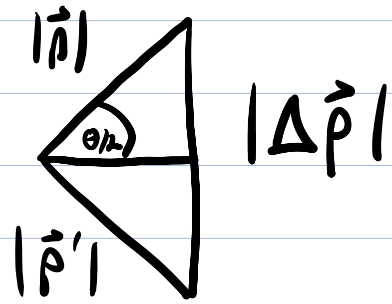
We derive the formula for the angular deflection of light, assuming it feels force

$$\vec{F} = -\frac{GME}{r^3}\hat{r}$$

We suppose the light comes in with momentum $\vec{p}$ and leaves with momentum $\vec{p}\,'$ and makes its closest approach at distance b.



By conservation of energy, at infinity, we know that $|\vec{p}| = |\vec{p}\,'|$. Thus, if we label the deflection angle θ, we have an isosceles triangle:



and thus we have the relationship

$$|\Delta \vec{p}| = 2p \sin\theta/2.$$

This is relevant as we can compute $\Delta\vec{p}$ another way, namely,

$$\Delta\vec{p} = \int \vec{F}\, dt$$

Based on our diagram, we can see that the vertical components of the force will cancel in the integral for the impulse. Thus, the integral reduces to

$$|\Delta\vec{p}| = \int \frac{GME}{r(\phi)^2}\cos\phi\, dt = GME\int_{-\phi_{max}}^{\phi_{max}} \frac{\cos\phi}{r(\phi)^2}\frac{1}{\dot{\phi}}\, d\phi.$$

Next, we can use conservation of angular momentum to substitute

$$l = E\, r(\phi)^2\dot{\phi} = bp$$

Where l is the angular momentum, and bp is the angular momentum on shortest approach. Performing this substitution, we have

$$|\Delta\vec{p}| = GME\int_{-\phi_{max}}^{\phi_{max}} \frac{\cos(\phi)}{r(\phi)^2}\cdot\frac{Er(\phi)^2}{bp} = 2\frac{GME^2}{bp}\sin\phi_{max}.$$

Next, from the diagram, we can see that the deflection angle θ is given by

$$2\pi - 2\phi_{max} + \theta = \pi \iff$$

$$\theta = 2\phi_{max} - \pi \implies$$

$$|\Delta \vec{p}| = \cancel{2}\,\frac{GME^2}{bp}\cos\theta/2 = \cancel{2}\,p\sin\theta/2 \implies$$

$$\tan\theta/2 = \frac{GME^2}{bp^2} \implies \theta = 2\arctan\frac{GME^2}{bp^2}.$$

We now use $E = p$, replace factors of c, and Taylor expand:

$$\boxed{\theta \approx \frac{2GM}{bc^2}}$$