We argue that there cannot be a phase transition in the 1D Ising model.

The Hamiltonian for the 1D Ising model is given by

$$H = -J \sum_{i=1}^{N} S_i^z S_{i+1}^z$$

where $S_j^z \in \{-\frac{1}{2}, \frac{1}{2}\}$ and $J > 0$. Clearly, the lowest energy state occurs when $S_i^z \cdot S_{i+1}^z = 1$ for all i. However, at finite temperature, the system minimizes its <u>free</u> energy.

The energy cost to flip one spin is $J/2$, which we can see by subtracting the energy of the one-flipped case from the all-aligned case. This energy cost is the same for a line of spins of any length.

We now compute the entropy. The number of microstates with one spin flipped is N, so the entropy is $k_B \log N$. So, for long chains at finite temperature,

$$\Delta F = \Delta E - T\Delta S = J/2 - T k_B \log N \to -\infty$$

as $N \to \infty$. Thus, the free energy can be significantly lowered upon the formation of a defect, so, thermodynamically, we do not expect order to form in the 1D Ising model at finite temperature.