We derive the value of the following series:

$$1-\tfrac{1}{2}+\tfrac{1}{3}-\tfrac{1}{4}+\tfrac{1}{5}-\cdots=\sum_{n=0}^{\infty}(-1)^n\frac{1}{n+1}$$

First, we compute the Taylor expansion for $\log(1+x)$ near $x=0$:

$$\log(1+x)\approx\log 1+\frac{1}{(1+x)^1}\bigg|_{x=0}\frac{x^1}{1!}-\frac{1}{(1+x)^2}\bigg|_{x=0}\frac{x^2}{2!}+$$

$$\frac{2}{(1+x)^3}\bigg|_{x=0}\frac{x^3}{3!}-\frac{6}{(1+x)^4}\bigg|_{x=0}\frac{x^4}{4!}+\frac{24}{(1+x)^5}\bigg|_{x=0}\frac{x^5}{5!}-\cdots$$

$$=x-\frac{1!}{2!}x^2+\frac{2!}{3!}x^3-\frac{3!}{4!}x^4+\frac{4!}{5!}x^5-\cdots=$$

$$x-x^2/2+x^3/3-x^4/4+x^5/5-\cdots=\sum_{n=0}^{\infty}(-1)^n\frac{x^{n+1}}{n+1}.$$

We can now clearly see that our initial series, $1-\tfrac{1}{2}+\tfrac{1}{3}-\tfrac{1}{4}+\tfrac{1}{5}-\cdots$, can be computed by simply plugging in $x=1$ above. Thus, we conclude

$$1-\tfrac{1}{2}+\tfrac{1}{3}-\tfrac{1}{4}+\tfrac{1}{5}-\tfrac{1}{6}+\cdots=\log(1+1)=\boxed{\log 2}$$