Exercise: We derive the Shannon entropy in the canonical ensemble.

Suppose we have a system with energy levels E_i at temperature T. In the canonical ensemble, the probability that the system is in state E_i is

$$p_i = e^{-E_i/k_BT} \cdot \frac{1}{Z}, \quad \text{where} \quad Z = \sum_i e^{-E_i/k_BT}.$$

We can also rearrange the first equation to express

$$E_i = -k_BT(\ln Z + \ln p_i).$$

We recall that the entropy is given by $S = -\frac{\partial F}{\partial T}$, where

$$F = -k_BT \ln Z.$$

We now compute the entropy:

$$S = -\frac{\partial F}{\partial T} = k_B \ln Z + \frac{k_BT}{Z}\frac{\partial Z}{\partial T} =$$

$$k_B \ln Z + \frac{k_B T}{Z} \sum_i e^{-E_i/k_B T} \cdot \frac{E_i}{k_B T^2} = k_B \ln Z + \frac{1}{T} \sum_i E_i \rho_i$$

$$= k_B \ln Z - k_B \sum_i \rho_i \ln \rho_i - k_B \sum_i \rho_i \ln Z \quad = -k_B \sum_i \rho_i \ln \rho_i$$

$$\longrightarrow \boxed{S = -k_B \sum_i \rho_i \ln \rho_i}$$