

We show that solutions to the Dirac equation

$$(i\not{\partial} - m)\Psi = 0$$

satisfy the Klein-Gordon equation.

It follows from the Dirac equation that

$$(i\not{\partial} + m)(i\not{\partial} - m)\Psi = 0 = (-\partial_\mu \gamma^\mu \partial_\nu \gamma^\nu + i\not{\partial}m - i\not{\partial}m - m^2)\Psi = \left(-\frac{1}{2} \partial_\mu \partial_\nu (\{\gamma^\mu, \gamma^\nu\} + [\gamma^\mu, \gamma^\nu]) - m^2\right)\Psi.$$

$\partial_\mu \partial_\nu [\gamma^\mu, \gamma^\nu] = 0$ since $\partial_\mu \partial_\nu$ is symmetric and the commutator is antisymmetric.

Since $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$, we are left with

$$(-\eta^{\mu\nu} \partial_\mu \partial_\nu - m^2)\Psi = (-\partial_\mu \partial^\mu - m^2)\Psi = 0,$$

which we recognize as the Klein-Gordon equation.