

Thermal de Broglie Wavelength from the Equipartition Theorem

Matt Kafker

Exercise:

We derive the thermal de Broglie wavelength using the equipartition theorem.

Solution:

The de Broglie wavelength of a particle with momentum p is given by

$$\lambda = \frac{2\pi\hbar}{p}.$$

The equipartition theorem says that, on average, each degree of freedom in a system at temperature T has energy $\frac{1}{2}k_B T$. Thus, a free particle in a system in thermal equilibrium should obey

$$T = \frac{\bar{p}^2}{2m} = \frac{3}{2}k_B T \implies p = \sqrt{3mk_B T}$$

and thus the thermal de Broglie wavelength is given by

$$\lambda_{\text{thermal}} = \frac{2\pi\hbar}{\sqrt{3mk_B T}}.$$