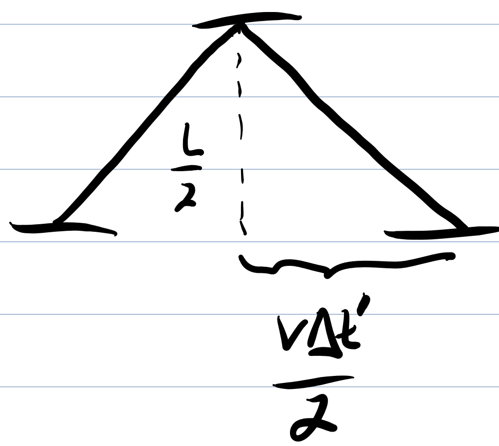
We derive the formula for time dilation.

We consider two observers, A and B. A is at rest and B is moving to the right with some speed v.

A is carrying a light clock, which tracks time by measuring the duration elapsed for a light pulse to travel back and forth between two mirrors, which are separated by a distance $L/2$.

A measures one such duration to be $\Delta t = L/c$. To B, A has travelled $-v\Delta t'$, so B determines that one "tick" lasts



$$\Delta t' = \frac{2}{c}\sqrt{\frac{v^2\Delta t'^2}{4} + \frac{L^2}{4}} = \sqrt{\frac{v^2}{c^2}(\Delta t')^2 + \frac{L^2}{c^2}} \implies$$

$$(\Delta t')^2\left(1 - v^2/c^2\right) = L^2/c^2 \implies$$

$$\Delta t' = \frac{L/c}{\sqrt{1 - v^2/c^2}} = \frac{\Delta t}{\sqrt{1 - v^2/c^2}} \equiv \gamma \Delta t \implies \boxed{\Delta t' = \gamma \Delta t}$$

(where we have used the fact that all observers measure c to be identical.)

This is the time dilation formula. B measures one "tick" to last longer than A does by a factor of γ.

Now, suppose that B is carrying an extended object of length λ' (measured with a ruler in the rest frame of B). A measures the object as it goes past by timing the duration between when the front end passes and the back end passes. This takes time T, so the object has length $\lambda = cT = cT'/\gamma = \lambda'/\gamma$.

$$\boxed{\lambda = \lambda'/\gamma}$$

This is the length contraction formula, which says that the object will appear contracted in A relative to how it appears to B.