Template

Matt Kafker

Exercise:

We derive the time independent Schrödinger equation.

Solution:

Consider the time-dependent Schrödinger equation with a potential $V(x)$ that does not depend on time,

$$i\hbar\dot{\Psi} = -\frac{\hbar^2}{2m}\Psi_{xx} + V\Psi.$$

Consider product solutions of the form $\Psi(x,t) = \psi(x)\phi(t)$. Plugging in, we have

$$i\hbar\psi\dot{\phi} = -\frac{\hbar^2}{2m}\psi_{xx}\phi + V\psi\phi \implies i\hbar\frac{\dot{\phi}}{\phi} = -\frac{\hbar^2}{2m}\frac{\psi_{xx}}{\psi} + V.$$

Now, the right hand side depends only on x and the left hand side depends only on t, which can only happen if each is constant. Call the constant of separation "E." Then, we have

$$i\hbar\dot{\phi(t)} = E\phi(t),$$
$$\boxed{-\frac{\hbar^2}{2m}\psi_{xx}(x) + V(x)\psi(x) = E\psi(x).}$$

The first equation determines the time-dependent phase of the product solution, and the second equation is the time-independent Schrödinger equation.