We show that the commutator of the raising and lowering operators of the quantum harmonic oscillator equals 1:

Let $a_\pm = \frac{1}{\sqrt{2m\hbar\omega}}(m\omega x \mp ip)$

Then, for some arbitrary test function f, we have

$$[a_-, a_+]f = (a_-a_+ - a_+a_-)f = \frac{1}{2m\hbar\omega}\Big[(m\omega x + ip)(m\omega x - ip)$$

$$-(m\omega x - ip)(m\omega x + ip)\Big]f = \frac{1}{2m\hbar\omega}\Big[(m\omega x)^2 + p^2 - im\omega(xp - px)$$

$$-\left((m\omega x)^2 + p^2 + im\omega(xp - px)\right)\Big]f = \frac{1}{2m\hbar\omega} \cdot -2im\omega[x, p]f$$

$$= \frac{-i}{\hbar} \cdot i\hbar f = f \Rightarrow [a_-, a_+] = 1. \quad \boxed{/\!/}$$